The SCS Curve Number Catchment in OpenHydroQual

Formulation, unit hydrograph routing, and reference

Abstract

The SCS (NRCS) Curve Number method is an event-based, cumulative procedure: it maps total rainfall onto total runoff and says nothing about rates. This document sets out how it is recast as a system of ordinary differential equations so that it can live inside a continuous simulator, shows that the recast form reproduces the original algebraic method exactly, describes the routing that turns runoff volume into an outlet hydrograph, and documents the resulting OpenHydroQual component and its parameters.

Contents

1 The classical method

For a storm of cumulative depth P, the curve number method gives cumulative direct runoff

$$Q = \frac{(P - I_a)^2}{P - I_a + S}, \qquad P > I_a, \tag{1}$$

and $Q = 0$ otherwise, where S is the maximum potential retention after runoff begins and I_a the initial abstraction. S follows from the curve number,

$$S = \frac{25400}{CN} - 254 \;\; [\mathrm{mm}] \qquad \Longleftrightarrow \qquad S = \frac{25.4}{CN} - 0.254 \;\; [\mathrm{m}], \tag{2}$$

and the standard closure is $I_a = 0.2\,S$. Writing $P_e = P - I_a$ for effective rainfall, the water not becoming runoff is the continuing retention

$$F = P_e - Q = \frac{S\,P_e}{P_e + S}. \tag{3}$$

Equation (1) is a relation between two accumulated depths. It has no time in it, which is why the method on its own produces a runoff *volume* and requires a separate unit hydrograph to produce a *hydrograph*.

2 Differential formulation

2.1 A single state

Let W be the *total* abstraction depth — the initial abstraction plus the continuing retention — so that $W = I_a + F$ and $W \to I_a + S = 1.2\,S$ as the storm grows. Define the runoff fraction

$$\boxed{C(W) = \left[1 - \left(\frac{S + I_a - W}{S}\right)^2\right]^+} \tag{4}$$

where $[\,\cdot\,]^+ = \max(\cdot, 0)$. The catchment is then two coupled reservoirs driven by the rainfall rate $i(t)$ over an area A:

$$\frac{dV_W}{dt} = i\,A\,(1 - C(W)) - k\,V_W, \qquad \frac{dV_1}{dt} = i\,A\,C(W) - \frac{V_1}{K}, \qquad W = \frac{V_W}{A}. \tag{5}$$

V_W is the abstraction store, V_1 the surface store, and k a drying coefficient discussed in section 2.3. Rainfall is partitioned at every instant: a fraction C becomes runoff, the rest is abstracted.

2.2 Why this is the classical method, exactly

Two properties of (4) do the work.

The initial abstraction is automatic. While $W < I_a$ the quantity $(S + I_a - W)/S$ exceeds unity, the bracket in (4) is negative, and the clamp gives $C = 0$. *No runoff whatever is produced until* $W = I_a$, which is precisely the meaning of the initial abstraction. From (5) with $C = 0$ we get $dW/dt = i$, so all rainfall is abstracted until the threshold is reached, and $W = P$ over that period. The threshold is therefore crossed exactly when $P = I_a$.

Beyond the threshold, the curve number equation is recovered. For $W > I_a$, ignoring drying and dividing (5) by i,

$$\frac{dW}{dP} = 1 - C = \left(\frac{S + I_a - W}{S}\right)^2. \tag{6}$$

Substituting $u = S + I_a - W$, so that $du/dP = -u^2/S^2$, and integrating from the threshold ($W = I_a$, $u = S$ at $P = I_a$):

$$\frac{1}{u} - \frac{1}{S} = \frac{P - I_a}{S^2} \quad \Longrightarrow \quad u = \frac{S^2}{P_e + S} \quad \Longrightarrow \quad W = I_a + \frac{S\,P_e}{P_e + S}, \tag{7}$$

which is $I_a + F$ with F exactly as in (3). The cumulative runoff is then

$$Q = P - W = (I_a + P_e) - I_a - \frac{S\,P_e}{P_e + S} = \frac{P_e^2}{P_e + S}, \tag{8}$$

identical to (1). The differential form is a reformulation, not an approximation.

Numerical behaviour. C is continuous, rising from zero with slope $2/S$ at $W = I_a$, so there is no discontinuous switch for the solver. The formulation is also self-limiting: $C \to 1$ only as $W \to 1.2S$, and the retention rate $1 - C$ vanishes there, so W approaches its bound asymptotically and never needs clipping.

2.3 Recovery between storms

The curve number method is an event method. Without recovery, W only increases, $C \to 1$, and the catchment becomes permanently impervious after enough cumulative rain. The $-k\,V_W$ term in (5) empties the abstraction store between events with time constant $1/k$, which should lie between a typical storm duration and a typical inter-event time. Water removed this way leaves the model; if it is needed downstream, the abstraction store should instead discharge to a soil or groundwater block.

3 The outlet hydrograph

3.1 Volume versus shape

Equation (5) determines how much water becomes runoff, and does so exactly. It says nothing about *when* that water arrives at the outlet. Standard practice convolves the excess-rainfall series with the NRCS dimensionless unit hydrograph. A continuous simulator does not convolve — it routes — and the routing is what fixes the shape.

3.2 Nash cascade

Runoff is routed through n equal linear reservoirs in series,

$$\frac{dV_j}{dt} = \frac{V_{j-1}}{K} - \frac{V_j}{K} \quad (j = 2 \ldots n), \qquad q_{out} = \frac{V_n}{K}, \tag{9}$$

the first of which is the surface store of (5). The impulse response of this chain is the gamma density

$$h(t) = \frac{1}{K\,\Gamma(n)} \left(\frac{t}{K}\right)^{n-1} e^{-t/K}, \tag{10}$$

which is the continuous-time counterpart of a gamma unit hydrograph. Its time to peak is $(n-1)K$.

3.3 Matching the NRCS unit hydrograph

Timing. Set

$$\boxed{K = \frac{L}{n-1}} \tag{11}$$

where L is the NRCS lag. The peak of the response then falls exactly at the lag, for any n.

It is worth being explicit about a trap here. The familiar expression $T_p = D/2 + 0.6\,T_c$ is the time to peak of a unit hydrograph of *finite* duration D. A continuous model convolves the *instantaneous* unit hydrograph, for which $D \to 0$ and the correct target is the lag alone, $L = 0.6\,T_c$. Including the $D/2$ term would shift every hydrograph late by half a rainfall interval.

Shape. With the timing fixed, n controls only how sharp the peak is. The dimensionless peak of the gamma response, $h_{max}T_p$, is $(n-1)^n e^{-(n-1)}/\Gamma(n)$; the NRCS dimensionless unit hydrograph with its peak rate factor of 484 corresponds to approximately 0.75:

n	3	4	**5**	6	NRCS target
$h_{max}T_p$	0.54	0.67	**0.78**	0.87	≈ 0.75

so $n \approx 4.7$, and $n = 5$ is the closest integer. A caution on the literature: values near $n \approx 3.7$ are also quoted for the "SCS-equivalent" gamma hydrograph. That figure matches the fraction of volume under the rising limb rather than the peak ordinate. The component here matches the peak; $n = 3$ is provided as a lighter alternative with a flatter peak and a longer recession.

3.4 Lag from catchment properties

The NRCS lag equation is used, since it already involves the curve number and is therefore consistent with the rest of the component:

$$L_{hr} = \frac{\ell^{0.8}\,(S'+1)^{0.7}}{1900\,\sqrt{Y}}, \qquad \ell \text{ [ft]}, \quad S' = \frac{1000}{CN} - 10 \text{ [in]}, \quad Y \text{ [\%]}. \tag{12}$$

Converting to metres and days, and writing the slope s as a dimensionless rise over run ($Y = 100\,s$):

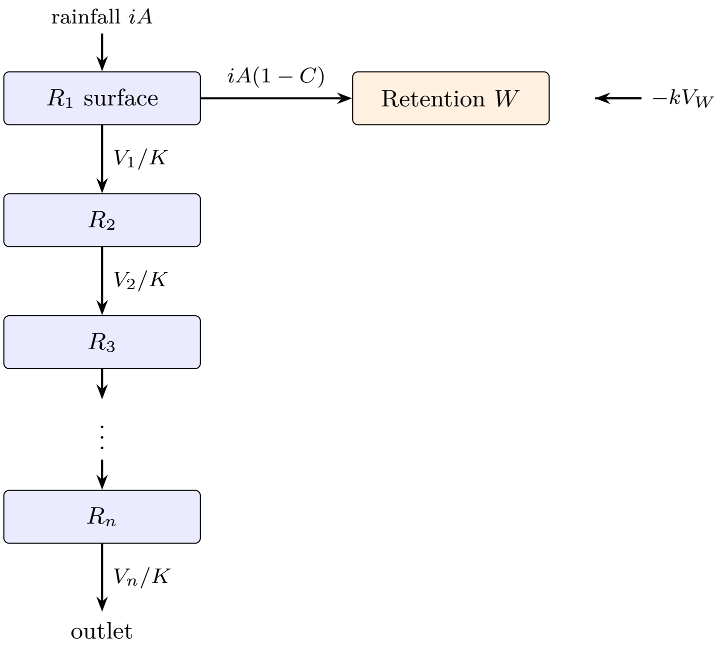
$$\boxed{L_{day} = \frac{\ell_m^{0.8}\left(\dfrac{1000}{CN} - 9\right)^{0.7}}{176300\,\sqrt{s}}} \qquad T_c = \frac{L}{0.6}. \tag{13}$$

The equation is calibrated for catchments below roughly 800 ha with CN between about 50 and 95; outside that range the lag should be supplied by other means. Note that only L is needed, since $K = L/(n-1)$; the time of concentration is a diagnostic.

4 Implementation

4.1 Structure

The component is a composite of $n+1$ blocks. Rainfall falls on the surface reservoir R_1; the partition link removes the abstracted fraction to the retention store; the remainder cascades through $R_1 \ldots R_n$ and leaves through the outlet link.



4.2 Types

All are defined in `resources/cn_catchment.json`.

Type	Kind	Role
`CN_retention`	block	Abstraction store V_W. Carries CN, computes S, I_a, W and the runoff fraction C, and applies the drying loss.
`CN_reservoir`	block	One linear reservoir of the cascade; discharge V/K. The first also receives the rainfall.
`CN_partition_link`	link	Carries $i\,A\,(1-C)$ from the surface reservoir to the retention store. Flow expression `Precipitation.s*(1-C.e)`.
`CN_cascade_link`	link	Discharge `Storage.s/K.s` between successive reservoirs.
`CN_outlet`	link	The outlet hydrograph, `Storage.s/K.s` from the last reservoir. The only way water leaves the component.
`CN_Catchment`	composite	$n = 5$, matched to the NRCS peak.
`CN_Catchment_3`	composite	$n = 3$, lighter, flatter peak.

The partition happens in a *link* rather than a block because the runoff fraction depends on the rainfall rate, held by the surface reservoir, and on W, held by the retention store. A link expression can see both endpoints; a block expression cannot reach another block.

4.3 Ports

External connections resolve by link type:

Link type	Direction	Attaches to
`CN_outlet`	out of the composite	the last reservoir R_n
`CN_outlet`	into the composite	the surface reservoir R_1
any other ($*$)	into the composite	the surface reservoir R_1

Because the fallback offers no outward direction, `CN_outlet` is the only way water leaves a catchment. It attaches inbound as well, so the outlet of one catchment may discharge directly into the next one downstream. Inflow of any kind enters the surface reservoir and is routed onward without being subjected to the curve number partition, which applies to rainfall only — water arriving from upstream has already had its abstraction taken out.

4.4 Parameters

Property	Units	Notes
`CN`	—	Curve number, usable range about 30–98. Sets S and $I_a = 0.2S$ and enters the lag equation. $S \to 0$ at $CN = 100$.
`area`	m^2, ft^2, km^2	Applied to every member.
`hydraulic_length`	m, ft, km, yd	Longest flow path ℓ; with CN and slope it sets L and hence every $K = L/(n-1)$.
`slope`	—	Dimensionless rise over run; converted to percent internally for the lag equation.
`recovery_coefficient`	1/day, 1/hr	Drying rate k.
`initial_abstraction_depth`	m, mm, in, ft	Antecedent condition; zero means the first $0.2S$ of rain yields no runoff.
`Precipitation`	—	Precipitation source, applied to R_1.

All values are held internally in metres and days. Outflow is reported with the usual flow units, `m^3/day;ft^3/s;m^3/s`.

4.5 Outputs

Useful reported quantities include, on the retention member, S, I_a, W, C and the drying loss; on each reservoir, the stored volume, equivalent depth and discharge; and on the outlet link, the flow, which is the hydrograph.

5 Verification

A uniform design storm of 240 mm over 24 hours on a 10 ha catchment with $CN = 75$ and drying disabled, so that the analytical result is unambiguous. Here $S = 84.7$ mm, $I_a = 16.9$ mm and $P_e = 223.1$ mm, giving $Q = 161.7$ mm from (1).

	simulated	**analytical**	**difference**
abstraction	7826.4 m^3	7830.6 m^3	0.05%
runoff	16185.6 m^3	16169.4 m^3	0.10%
total (mass balance)	24012.0 m^3	24000.0 m^3	0.05%

The residuals are at the level of the solver tolerance. The residence time was also checked: for $\ell = 1000$ m, $CN = 75$ and $s = 0.02$ the lag equation gives $L = 0.02812$ day (0.675 hr) and the model assigned $K = 0.007030$ day $= L/4$.

Not yet verified. The peak timing and shape of the hydrograph follow by construction from $K = L/(n-1)$, but have not been confirmed numerically. That check is best done by inspecting the outlet flow series in the interface.

6 Worked usage

```
create source;type=Precipitation,name=Rain,timeseries=precipitation.csv
create composite;type=CN_Catchment,name=Cat1,x=200,y=0,
    CN=75,area=100000[m~^2],hydraulic_length=1000[m],slope=0.02,
    recovery_coefficient=0.1[1/day],initial_abstraction_depth=0,Precipitation=Rain
create link;from=Cat1,to=Outlet,type=CN_outlet,name=Outflow
```

Catchments may be chained: a `CN_outlet` from one may discharge into another, whose surface reservoir routes the incoming flow onward.

A complete model is in `Examples/CN_Catchment/CN_watershed.ohq`: a rural catchment discharging into an urban one and on to the watershed outlet, alongside a third catchment that repeats the rural parameters with the three-reservoir variant so the two cascades can be compared on one plot. The folder `Examples/CN_Catchment` also holds the design storm used to drive it.

7 Assumptions and limitations

- **Curve number closure.** $I_a = 0.2S$ is assumed. Studies favouring $\lambda = 0.05$ would require a different closure and a re-fitted S.
- **Recovery is empirical.** The exponential drying term has no basis in the curve number method itself; it is the device that lets an event method run continuously, and k is a calibration parameter. Recovered water leaves the model.
- **Antecedent moisture** is represented only through the state of the abstraction store. There is no AMC I/II/III switching.
- **Shape versus peak.** The cascade matches the NRCS peak ordinate with $n = 5$; it is not a term-by-term reproduction of the dimensionless hydrograph ordinates.
- **The lag equation** is calibrated for catchments below roughly 800 ha with CN between 50 and 95.
- **No baseflow, no channel routing, no snow.** The component produces direct runoff from a single catchment only.
- **Uniform rainfall** over the catchment is assumed, as in the original method.