Culvert Modelling in OpenHydroQual

HDS-5 inlet control, outlet control, and storage-routed barrels

Abstract

A culvert is not a pipe with a friction law. Its discharge is set by whichever of two independent mechanisms is more restrictive: the geometry of the entrance, which can pass only so much flow for a given headwater regardless of what happens downstream, or the energy budget along the barrel, which depends on tailwater. This document sets out how both mechanisms are formulated for OpenHydroQual, how the FHWA HDS-5 inlet-control equations are inverted so that discharge can be obtained explicitly from head, how the barrel is represented as a storage-routed element between two open channels, and what the resulting components do and do not claim to reproduce.

Contents

1 Scope and conventions

The components documented here live in `resources/culvert.json` and are registered as the *Culverts* template in `resources/default_templates.list`. Worked models are in `Examples/Culvert`.

1.1 Units

OpenHydroQual solves internally in metres and days. Every discharge expression in the templates therefore carries a factor of 86400 converting $\mathrm{m^3\,s^{-1}}$ to $\mathrm{m^3\,day^{-1}}$, and every occurrence of $2g$ appears as the numeric constant $19.62\ \mathrm{m\,s^{-2}}$. Elevations, inverts and heads share a single vertical datum with the connected channel blocks.

1.2 What the expression language permits

The formulation below is shaped by a hard constraint: quantities are declared as algebraic expressions evaluated by `aquifolium/src/ExpressionNode.cpp`, and that evaluator offers no conditionals, no loops, and no implicit solves. Everything must be written as a single explicit algebraic expression in terms of other quantities. The available operations that matter here are

- `_min`, `_max` — binary only, so a three-way selection must be nested;
- `_pos`$(x) = \max(x, 0)$ and `_hsd`$(x) = H(x)$, a hard step;
- `_sqr`$(x) = \sqrt{\max(x, 0)}$, a magnitude-only square root;
- `_sqt`$(x) = \mathrm{sgn}(x)\sqrt{x^2/(|x| + 10^{-4})}$, a *signed, regularised* square root;
- `_mon`$(a, b) = a/(a + b)$, a smooth saturating switch;
- the power operator, evaluated as $\mathrm{pow}(|a|, b)$, so the exponent may itself be an expression.

`_sqt` is the workhorse. A culvert must pass flow in either direction and must remain differentiable as the driving head crosses zero, and $\mathrm{sgn}(\Delta H)\sqrt{|\Delta H|}$ is not differentiable there. The regularised form behaves as $\sqrt{\Delta H}$ away from the origin and as a smooth ramp through it.

2 Barrel geometry

2.1 Circular section

For a circular barrel of diameter D flowing at depth d, write $x = d/D$. The templates use the polynomial fits already established elsewhere in the OpenHydroQual component library (they appear in `Sewer_pipe`), so that a culvert and a sewer conduit describe the same section the same way:

$$A(x) = D^2 \left(-0.910331972\,x^3 + 1.376045712\,x^2 + 0.319684423\,x\right), \tag{1}$$

$$P(x) = D \left(5.486583\,x^3 - 8.60202\,x^2 + 6.1081605\,x\right). \tag{2}$$

Equation (1) is accurate: at $x = 1$ it returns $0.785398\,D^2$, which is $\pi D^2/4$ to six figures. Equation (2) is not: at $x = 1$ it returns $2.9925\,D$ against the exact $\pi D = 3.14159\,D$, an underestimate of 4.8%. This propagates into the hydraulic radius and hence into the friction term of outlet control; see Section 12.

2.2 Recovering depth from storage

The storage-routed barrel of Section 6 carries volume as its state and must recover depth from it. This inverts (1), which is cubic in x and has no convenient closed-form inverse. Writing $u = A/A_{\text{full}}$ for the fraction of the section that is wetted, the templates use a quintic fit constrained to pass through both endpoints exactly:

$$\frac{d}{D} = 1.9470734\, u - 4.948471\, u^2 + 10.344915\, u^3 - 10.5243768\, u^4 + 4.1808594\, u^5. \tag{3}$$

This is monotone on $u \in [0,1]$, exact at $u = 0$ and $u = 1$, and its worst error against a numerical inversion of (1) is $0.005\, D$, half a percent of the diameter.

2.3 Box section

For a rectangular barrel of span B and rise H the area is exact, $A = Bd$, and the depth inverts trivially. The wetted perimeter must grow from B at $d = 0$ to $2(B + H)$ when the barrel runs full, while not counting the soffit before the water reaches it. The templates use

$$P = B\left(1 + \left(\frac{d}{H}\right)^4\right) + 2d, \tag{4}$$

which is exact at both ends and engages the soffit only as the barrel approaches full: at half depth it gives $1.0625B + H$ against the free-surface value $B + H$.

3 Inlet control

3.1 The HDS-5 equations

FHWA HDS-5 expresses inlet control as a relation between headwater and discharge through the dimensionless discharge parameter

$$q^* = \frac{K_u\, Q}{A\, D^{0.5}}, \qquad K_u = 1.811 \text{ (SI)}, \tag{5}$$

in which A is the *full* barrel area and D the barrel rise. There are three forms. Unsubmerged Form (1) adds the specific head at critical depth H_c; unsubmerged Form (2) omits it; the submerged form is a quadratic:

$$\text{Form (1):} \quad \frac{HW_i}{D} = \frac{H_c}{D} + K\,(q^*)^M + K_s S, \tag{6}$$

$$\text{Form (2):} \quad \frac{HW_i}{D} = K\,(q^*)^M, \tag{7}$$

$$\text{submerged:} \quad \frac{HW_i}{D} = c\,(q^*)^2 + Y + K_s S. \tag{8}$$

S is the barrel slope and K, M, c, Y are tabulated in HDS-5 Table A.1 (reproduced in Section 10) by barrel shape, material and inlet edge treatment. The slope correction is $K_s = -0.5$ for all inlets except mitered ones, which take $K_s = +0.7$. Note that Form (2) carries *no* slope term.

Rearranging Form (1) to isolate the two discharge-dependent terms gives $H_c/D + K(q^*)^M = HW_i/D - K_s S$, so the templates work with $\tau = HW_i/D + \lambda S$ where $\lambda = -K_s$. The parameter `slope_correction_coeff` is therefore entered as 0.5 in the normal case and -0.7 for a mitered inlet.

The transition limits quoted by HDS-5 — unsubmerged up to $Q/AD^{0.5} = 3.5$ (1.93 SI), submerged above 4.0 (2.21 SI) — are two statements of the same threshold on the dimensionless group q^*, since $1.811 \times 1.93 = 3.50$ and $1.811 \times 2.21 = 4.00$. Comparing q^* against 3.5 and 4.0 is thus unit-independent and is what the templates do.

3.2 Inverting for discharge

HDS-5 is written to answer "what headwater does this discharge require?". OpenHydroQual needs the opposite: the solver knows heads and must produce a flow. Two of the three forms invert directly. From (8),

$$q^*_{\text{sub}} = \sqrt{\frac{HW/D - Y + 0.5S}{c}}, \tag{9}$$

and from (7), $q^*_{(2)} = \left(\frac{HW/D}{K}\right)^{1/M}$. Both are single expressions. Form (1) is the difficult one, because H_c is the specific head at critical depth *for the discharge being sought*, making (6) implicit.

3.3 The critical-flow function

Resolving Form (1) needs the map from a critical specific head to the discharge that produces it. For a circular section at critical depth d_c, with $\theta = 2\arccos(1 - 2d_c/D)$, top width T and area A_c,

$$Q_c = \sqrt{\frac{gA_c^3}{T}}, \qquad \frac{H_c}{D} = \frac{d_c}{D} + \frac{A_c}{2TD}, \tag{10}$$

which is a parametric curve in d_c. Eliminating d_c numerically and writing $\chi = H_c/D$, the templates use

$$\begin{aligned}\frac{Q_c}{D^{5/2}} = \chi^2 (1.9179351 - 0.4294827\,\chi - 0.0636513\,\chi^2 \\ + 0.0260040\,\chi^3 - 0.0431370\,\chi^4 + 0.0178421\,\chi^5),\end{aligned} \tag{11}$$

valid for $10^{-4} \leq \chi \leq 1.6$ with a maximum *relative* error of 0.007%. The χ^2 prefactor is not cosmetic: it carries the correct small-depth asymptote, and normalising instead by $\chi^{3/2}$ degrades the fit by three orders of magnitude. The diameter cancels, so (11) is a pure function of χ and $q^*_{\text{crit}}(\chi) = K_u\,(Q_c/D^{5/2})/(\pi/4)$.

For a box section the same construction is exact and the geometry cancels entirely. Critical flow gives $H_c = \frac{3}{2}d_c$ and $Q_c = B\sqrt{g\,d_c^3}$, whence

$$q^*_{\text{crit}}(\chi) = K_u\sqrt{g}\,\left[\min\left(\tfrac{2}{3}\chi,\,1\right)\right]^{3/2}. \tag{12}$$

3.4 Resolving Form (1)

With q^*_{crit} available, write $\tau = HW/D + \lambda S$ for the slope-corrected headwater ratio. Form (1) states $H_c/D = \tau - K(q^*)^M$, so the fixed point sought is

$$q^* = q^*_{\text{crit}}\left(\tau - K\,(q^*)^M\right). \tag{13}$$

Because $K(q^*)^M$ is a small correction to τ over the unsubmerged range, this converges — but the undamped map is expansive at large q^* and oscillates. The templates therefore unroll four *damped* iterations,

$$q_1^* = q_{\text{crit}}^*(\tau), \qquad q_{k+1}^* = \min\Big(\tfrac{1}{2}\Big[q_k^* + q_{\text{crit}}^*\Big(\tau - K(q_k^*)^M\Big)\Big],\ q_1^*\Big), \tag{14}$$

and take q_4^*. The clamp to q_1^* is a rigorous bound, not a safety hack: $K(q^*)^M \geq 0$ forces $H_c/D \leq \tau$, and q_{crit}^* is monotone, so the true root can never exceed the first iterate. The argument of q_{crit}^* is additionally clamped to the fitted range of (11).

3.5 The unsubmerged–submerged transition

HDS-5 applies the unsubmerged form below $q^* = 3.5$, the submerged form above $q^* = 4.0$, and interpolates between. The templates implement exactly that,

$$w = \min\left(\max\left(\frac{q_{\text{sub}}^* - 3.5}{0.5},\, 0\right),\, 1\right), \qquad q^* = (1-w)\, q_{\text{unsub}}^* + w\, q_{\text{sub}}^*. \tag{15}$$

The weight is keyed to q_{sub}^*, not to q_{unsub}^*. This matters. The submerged branch (9) is closed-form and monotone everywhere, whereas the iterated unsubmerged branch is only trustworthy inside its own regime. Keying the blend to the stable branch means the unsubmerged value contributes *exactly zero* above $q^* = 4$, where it is no longer meaningful. Keying it the other way — the obvious choice — lets a diverging iterate leak into the answer at high headwater.

Finally the discharge follows by inverting (5): $Q_{\text{inlet}} = N\, q^* A D^{0.5}/K_u$ for N barrels.

4 Outlet control

Outlet control is the energy budget from headwater to tailwater: an entrance loss, friction along the barrel, and a velocity-head loss at the exit. With k_e and k_x the entrance and exit loss coefficients, n Manning's coefficient, L the barrel length and R the hydraulic radius,

$$Q_{\text{outlet}} = A\, \texttt{\_sqt}\left(\frac{2g\,\Delta H}{k_e + k_x + 2gn^2 L\, R^{-4/3}}\right), \tag{16}$$

signed by ΔH through the regularised root. The driving head is measured between effective water surfaces referenced to the local inverts, $\Delta H = \max(h_s, z_{\text{in}}) - \max(h_e, z_{\text{out}})$, so that a tailwater below the outlet invert produces a free outfall rather than a spurious suction.

4.1 The $(d_c + D)/2$ grade line rule

Equation (16) as written references the tailwater to the outlet invert, which is wrong when the barrel is running full but the tailwater is low. HDS-5 Section 3.2.2 records that backwater calculations show the extended full-flow hydraulic grade line pierces the outlet plane midway between critical depth and the top of the barrel, and prescribes

$$h_o = \max\Big(TW,\ z_{\text{out}} + \tfrac{d_c + D}{2}\Big), \qquad d_c \leq D. \tag{17}$$

This needs critical depth for the discharge being sought, so like Form (1) it is implicit — but the feedback is negative (raising h_o lowers Q, which lowers d_c, which lowers h_o), so it contracts. Three unrolled corrections suffice. For a circular barrel, writing $\psi = Q/(\sqrt{g}D^{5/2})$ per barrel,

$$\frac{d_c}{D} = \sqrt{\psi}\,(0.9641769 + 0.3201129\,\psi - 0.835724\,\psi^2 \\ + 1.4329776\,\psi^3 - 1.4308019\,\psi^4 + 0.4947965\,\psi^5), \tag{18}$$

monotone with a worst error of $0.010\,D$, concentrated near $d_c/D \to 1$ where the cap $d_c \leq D$ takes over anyway. For a box the relation is exact, $d_c = (q^2/g)^{1/3}$ with q the discharge per unit span.

4.2 Why the rule must be gated, and gated widely

HDS-5 is explicit that (17) is an approximation with a limited range: it "can only be used if the barrel flows full for most of its length", "should not be used if the inlet is not submerged", and gives adequate results only down to a headwater of $0.75D$. Applying it unconditionally would impose a large spurious backwater on a nearly empty barrel. The templates therefore ramp it in with a weight

$$\gamma = \min\left(\max\left(\frac{(h_s - z_{\text{out}})/D - \gamma_{\text{lo}}}{\gamma_{\text{hi}} - \gamma_{\text{lo}}}, 0\right), 1\right), \qquad h_o^{\text{eff}} = TW + \gamma\,\texttt{\_pos}(h_o - TW)\,. \tag{19}$$

The upper limit is 1.8, not the 1.2 HDS-5 implies, and the reason is numerical. Over the ramp the grade line rises by up to $(d_c + D)/2 \approx 0.85D$ while the barrel head rises by only $(\gamma_{\text{hi}} - \gamma_{\text{lo}})D$. If the band is narrower than about 0.85, the driving head *falls* as the barrel fills: the outlet rating curve turns non-monotonic and the storage balance acquires up to three simultaneous steady states. Sweeping the band width confirms the fold and the extra roots vanish together at a width of 1.05. Both limits are exposed as `hgl_rule_lower` and `hgl_rule_upper`; narrowing the band towards HDS-5's nominal 1.2 reintroduces the multiple roots.

5 Combining the two controls

The governing discharge is the more restrictive of the two mechanisms. HDS-5 computes the headwater each requires at a given Q and takes the larger; the templates compute the capacity each permits at a given head and take the smaller. These are equivalent whenever both rating curves are monotone, which they are. Preserving the sign,

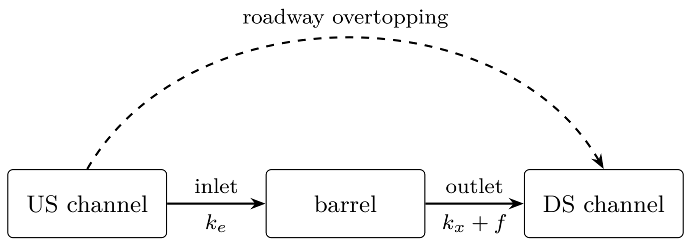
$$Q = \min(Q_{\text{inlet}}, |Q_{\text{outlet}}|) \cdot \frac{Q_{\text{outlet}}}{|Q_{\text{outlet}}| + \varepsilon}, \qquad \varepsilon = 10^{-6}, \tag{20}$$

which is continuous through $Q_{\text{outlet}} = 0$ and avoids sgn, whose discontinuity would otherwise sit exactly where the solver spends its time.

6 The storage-routed barrel

6.1 Layout

A culvert between two open channels is represented as three objects: an inlet transition link carrying inlet control and the entrance loss, a barrel block carrying volume, and an outlet transition link carrying barrel friction and the exit loss.



Splitting the losses across two links is not an approximation. At steady state the same Q passes both, so $\Delta H_1/k_e = \Delta H_2/(k_x + f)$ with $\Delta H_1 + \Delta H_2 = HW - TW$, giving $\Delta H_1 = k_e(HW - TW)/(k_e + k_x + f)$ and hence exactly (16). The split is verified numerically in Section 11.

The entrance-loss term on the inlet link uses the *full* barrel area, not the barrel's current wetted area. Using the current area deadlocks the model: an empty barrel has zero area, so the entrance-loss capacity is zero, so no water can ever enter. In the partially-full regime inlet control governs anyway, and in the full regime the full area is the correct one.

6.2 Surcharge and the Preissmann slot

Once the barrel runs full it can carry pressure head above its crown, but a free-surface storage element cannot represent that: storage saturates at $A_{\text{full}}L$ and stage stops responding. The templates use the standard device of a narrow virtual slot above the crown of width σD, so that

$$h = z_b + d(\mathcal{S}) + \frac{\max(\mathcal{S} - \mathcal{S}_{\text{full}}, 0)}{NL\sigma D}, \tag{21}$$

with σ exposed as `slot_width_ratio`. A narrow slot is more physically faithful and stiffer to solve; the default $\sigma = 0.02$ trades towards fidelity, and widening it is the first thing to try if a surcharging model struggles to converge.

7 Roadway overtopping and flow accounting

When headwater exceeds the road crest the embankment acts as a broad-crested weir. HDS-5 Equation 3.9 gives $Q_o = C_d L\, HW_r^{1.5}$ with $C_d = k_t C_r$, where C_r is the free-flow coefficient from Figure 3.11A/B and k_t the correction for downstream submergence from Figure 3.11C. HDS-5 also notes $C_d(\text{SI}) = 0.552\, C_d(\text{English})$, so the deep-overtopping value of about 3.1 in US units becomes the default $C_r = 1.7$ used here.

Because k_t is published as a graph rather than an equation, it is exposed as the input `road_submergence_factor` (default 1.0, a free overfall) rather than fitted, and the diagnostic `road_submergence_ratio` is written to the output so that the tailwater encroachment can be checked against Figure 3.11C. The two paths are computed separately and reported separately, so that the split between them is auditable:

$$Q_{\text{road}} = k_t C_r B_r \left(\texttt{\_pos}(h_s - z_r)^{3/2} - \texttt{\_pos}(h_e - z_r)^{3/2}\right), \tag{22}$$

signed, and continuous as either side crosses the crest. In the single-link variants the quantities `culvert_flow` (all barrels) and `overtopping_flow` are both written to the output, together with `overtopping_fraction`, and

$$\texttt{flow} = \texttt{culvert\_flow} + \texttt{overtopping\_flow} \tag{23}$$

is always the *total* — it is the quantity the storage balances aggregate. In the routed layout the roadway is a separate `Roadway_Overtopping` link and is therefore reported on its own.

8 Numerical considerations

Three choices in the templates exist purely to keep the implicit solver well-behaved, and are worth recording because they are not obvious from the hydraulics.

1. **Smooth switches over hard ones.** Regime blending uses `_mon` and clamped linear ramps rather than `_hsd`. A Heaviside step has zero derivative everywhere and a jump at the threshold, which stalls a Newton iteration sitting on it.

2. **Upwinding without sgn.** Where a quantity must be taken from the upstream side, the templates use the continuous weight $w = \frac{1}{2} + \frac{1}{2}\,\Delta H/(|\Delta H| + 10^{-3})$, which tends to 1 and 0 away from the origin and passes smoothly through it.

3. **Floors belong in denominators, never in flow terms.** An early version floored the outlet link's flow area at $10^{-4}\,\mathrm{m}^2$ to protect the velocity calculation. Because that area multiplies into the discharge, a drained barrel then leaked about $5 \times 10^{-6}\,\mathrm{m}^3\,\mathrm{s}^{-1}$ indefinitely and storage drifted negative. The area entering `flow` is now unfloored — it is legitimately zero when the barrel is dry — and the floor appears only in the denominator of `velocity`.

9 Components

9.1 Types

Type	**Kind**	**Role**
`Culvert Barrel (Circular)`	block	Storage-routed circular barrel; carries volume, stage, section geometry and the friction coefficient.
`Culvert Barrel (Box)`	block	As above for a rectangular section.
`Channel_to_culvert (Circular)`	link	Inlet transition: HDS-5 inlet control plus entrance loss.
`Channel_to_culvert (Box)`	link	As above, using the exact box critical-flow relation.
`Culvert_to_channel (Circular)`	link	Outlet transition: barrel friction, exit loss and the $(d_c + D)/2$ rule.
`Culvert_to_channel (Box)`	link	As above, using the exact rectangular critical-depth relation.
`Roadway_Overtopping`	link	Weir flow over the embankment, connecting the two channels directly.
`Circular Culvert`	link	Lumped single-link culvert: both controls in one connector, no barrel storage.
`Box Culvert`	link	As above for a rectangular section.

The two layouts are alternatives. The lumped link is appropriate for short culverts where barrel storage is negligible; the routed barrel adds attenuation and gives constituents a real residence time inside the structure. The outlet link and the roadway link are shape-generic because the barrel blocks expose a common vocabulary (`barrel_rise`, `barrel_area`, `inlet_crest_width`, `friction_loss_coeff`), so only the inlet link needs a per-shape variant. The outlet link became shape-specific only when the $(d_c + D)/2$ rule of Section 4.1 was added, since critical depth is a property of the section.

9.2 Barrel parameters

Parameter	**Meaning**	**Default**	**Unit**
`Number_of_barrels`	Number of barrels	1	–

Parameter	Meaning	Default	Unit
diameter	Barrel diameter (circular)	–	m
barrel_width	Span (box)	–	m
barrel_height	Rise (box)	–	m
length	Barrel length	–	m
inlet_invert	Invert elevation at the upstream face	–	m
outlet_invert	Invert elevation at the downstream face	–	m
ManningCoeff	Manning's roughness coefficient	0.013	–
inflow	Optional direct inflow time series	–	m^3/day
inlet_crest_width_factor	Legacy crest-width factor (circular)	0.65	–
slot_width_ratio	Preissmann slot width, as a fraction of the rise	0.02	–
depth	Initial water depth	–	m

9.3 Inlet link parameters

Parameter	Meaning	Default	Unit
entrance_loss_coeff	HDS-5 k_e	0.5	–
HDS5_K	Table A.1 coefficient K	0.0098	–
HDS5_M	Table A.1 exponent M	2.0	–
HDS5_c	Table A.1 coefficient c	0.0398	–
HDS5_Y	Table A.1 coefficient Y	0.67	–
HDS5_form1_switch	1 for equation Form (1), 0 for Form (2)	1	–
slope_correction_coeff	λ in $\tau = HW/D + \lambda S$; 0.5 normally, -0.7 mitered	0.5	–

The defaults are one row of Table A.1 each; change them to match your inlet. The circular templates default to Chart 1 scale 1 (square edge with headwall) and the box templates to Chart 8 scale 1 (30–75° wingwall flares). Every other configuration has its own constants *and possibly a different equation form* — note in Section 10 that all circular entries are Form (1) while box Charts 9–13 are Form (2), which requires `HDS5_form1_switch` = 0.

10 HDS-5 Table A.1 constants

Reproduced from HDS-5 Third Edition, Table A.1 (*Constants for Inlet Control Equations for Charts in Appendix G*), for the circular and rectangular shapes these components cover. Tapered-inlet and non-circular rows (Charts 55–59 and Tables A.2–A.6) are omitted, as are shapes the templates do not represent.

HDS-5 Table A.1 — constants for inlet control equations, circular and rectangular shapes.

Chart	Shape / Material	Scale	Inlet configuration	Form	K	M	c	Y
1	Circular Concrete	1	Square edge w/headwall	1	0.0098	2.0	0.0398	0.67
1	Circular Concrete	2	Groove end w/headwall	1	0.0018	2.0	0.0292	0.74
1	Circular Concrete	3	Groove end projecting	1	0.0045	2.0	0.0317	0.69
2	Circular CM	1	Headwall	1	0.0078	2.0	0.0379	0.69
2	Circular CM	2	Mitered to slope	1	0.0210	1.33	0.0463	0.75
2	Circular CM	3	Projecting	1	0.0340	1.50	0.0553	0.54
3	Circular	A	Beveled ring, 45° bevels	1	0.0018	2.50	0.0300	0.74
3	Circular	B	Beveled ring, 33.7° bevels	1	0.0018	2.50	0.0243	0.83
8	Rect. Box Concrete	1	30° to 75° wingwall flares	1	0.026	1.0	0.0347	0.81
8	Rect. Box Concrete	2	90° and 15° wingwall flares	1	0.061	0.75	0.0400	0.80
8	Rect. Box Concrete	3	0° wingwall flares	1	0.061	0.75	0.0423	0.82
9	Rect. Box Concrete	1	45° wingwall flare $d = .043D$	2	0.510	0.667	0.0309	0.80
9	Rect. Box Concrete	2	18° to 33.7° wingwall flare $d = .083D$	2	0.486	0.667	0.0249	0.83
10	Rect. Box Concrete	1	90° headwall w/ 3/4” chamfers	2	0.515	0.667	0.0375	0.79
10	Rect. Box Concrete	2	90° headwall w/ 45° bevels	2	0.495	0.667	0.0314	0.82
10	Rect. Box Concrete	3	90° headwall w/ 33.7° bevels	2	0.486	0.667	0.0252	0.865
11	Rect. Box Concrete	1	3/4” chamfers; 45° skewed headwall	2	0.545	0.667	0.04505	0.73
11	Rect. Box Concrete	2	3/4” chamfers; 30° skewed headwall	2	0.533	0.667	0.0425	0.705
11	Rect. Box Concrete	3	3/4” chamfers; 15° skewed headwall	2	0.522	0.667	0.0402	0.68
11	Rect. Box Concrete	4	45° bevels; 10–45° skewed headwall	2	0.498	0.667	0.0327	0.75
12	Rect. Box 3/4” chamf.	1	45° non-offset wingwall flares	2	0.497	0.667	0.0339	0.803
12	Rect. Box 3/4” chamf.	2	18.4° non-offset wingwall flares	2	0.493	0.667	0.0361	0.806
12	Rect. Box 3/4” chamf.	3	18.4° non-offset, 30° skewed barrel	2	0.495	0.667	0.0386	0.71
13	Rect. Box Top Bevel	1	45° wingwall flares – offset	2	0.497	0.667	0.0302	0.835
13	Rect. Box Top Bevel	2	33.7° wingwall flares – offset	2	0.495	0.667	0.0252	0.881
13	Rect. Box Top Bevel	3	18.4° wingwall flares – offset	2	0.493	0.667	0.0227	0.887

Two things in this table are easy to get wrong. First, the equation form is not a property of the shape: every circular row is Form (1), but among the boxes only Chart 8 is, and Charts 9–13 are Form (2). Second, Form (2) constants are numerically unlike Form (1) constants — $K \approx 0.5$ with $M = 0.667$ rather than $K \approx 0.03$ with $M \approx 1$–2 — because Form (2) absorbs the critical-depth term. Mixing a Form (2) constant set with `HDS5_form1_switch` $= 1$ produces a badly wrong rating rather than an error.

11 Verification

11.1 Round trip against the forward equations

The inversion of Section 3 was checked by choosing a discharge, evaluating the forward HDS-5 equations (6)–(8) to get HW/D using exact critical-depth geometry, then feeding that headwater back through the templates' inverse and comparing.

Case	Range	Worst error
Circular, Form (1)	$0.05 \leq Q \leq 5$ m^3/s	0.65%
Box, Form (1)	$0.5 \leq Q \leq 25$ m^3/s	1.04%
Submerged branch	$q^* > 4$	exact

The residual in the unsubmerged range is shared between the fit in (11) and the truncation of the damped iteration (14) at four steps.

11.2 End-to-end behaviour

The examples in `Examples/Culvert` exercise the components through the solver. Volume closes across the whole structure to between 0.021% and 0.153% in all ten. Two results are worth singling out:

- The lumped single link and the storage-routed barrel, driven by the same storm, agree to 0.05% on peak discharge and 0.01% on volume — confirming the loss-splitting argument of Section 6.
- Under a rising tailwater the governing mechanism genuinely changes hands: inlet control governs only 59% of the flowing steps and outlet control the remainder, rather than one mechanism dominating throughout.
- In the outlet-controlled example (a 60 m corrugated-metal barrel on a flat grade) the $(d_c+D)/2$ rule is active on 57% of the flowing steps and raises the effective tailwater from 0.45 to as much as 1.37 m, reducing outlet capacity by 10–23% relative to the uncorrected form.

The outlet links were additionally checked by evaluating the shipped expression strings directly, with the engine's own function semantics, against an independent implementation of Section 4.1: the two agree to machine precision, and the box variant's critical depth matches the exact rectangular relation.

12 Assumptions and limitations

1. **Wetted perimeter of a circular barrel is** 4.8% **low at full flow** (2). The fit is inherited from the existing conduit components for consistency. It inflates R by about 5% and the friction-controlled discharge by roughly 3%; it does not affect inlet control at all.

2. **No barrel water-surface profile.** The barrel is a single storage cell, so there are no M1/M2/S2 profiles and no hydraulic jump location.
3. **No normal-depth representation on steep slopes.** A steep culvert on inlet control physically runs at normal depth in the barrel; a lumped storage cell returns a single depth derived from volume instead.
4. **The $(d_c + D)/2$ rule is ramped over a wider band than HDS-5 implies**, for the reason given in Section 4.2. Between $0.75D$ and $1.8D$ of barrel head the rule is only partially applied, so the outlet-control capacity there sits between the corrected and uncorrected values.
5. **The Preissmann slot is a numerical device.** It has no counterpart in a conventional culvert calculation and slightly inflates stored volume once the barrel surcharges.

Items 2 and 3 are structural consequences of a lumped-storage barrel and would require a different element to address.

13 References

1. Federal Highway Administration, *Hydraulic Design of Highway Culverts*, Hydraulic Design Series No. 5, 3rd ed., FHWA-HIF-12-026, 2012. `https://www.fhwa.dot.gov/engineering/hydraulics/pubs/12026/hif12026.pdf`
2. Component definitions: `resources/culvert.json`.
3. Worked models and verification harness: `Examples/Culvert`.